ENTRANCE EXAMINATION FOR ADMISSION, MAY 2012.

Ph.D. (PHYSICS)

COURSE CODE : 122

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 122

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each of the question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

1. With which type of spectroscopy would one observe the pure rotation spectrum of H_2?

(A) Microwave (B) Raman (C) Ultraviolet (D) NMR

2. The vibrational frequency of a diatomic molecule of reduced mass μ and force constant k is expressed as

(A) $2\pi\sqrt{\mu k}$ (B) $\frac{1}{2\pi}\sqrt{\frac{\mu}{k}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$ (D) $\sqrt{\frac{k}{\mu}}$

3. The point group of benzene molecule is

(A) C_{2v} (B) D_{6h} (C) C_{3v} (D) D_{2h}

4. The difference between soft and hard X-rays lies in

(A) Velocity (B) Intensity (C) Polarization (D) Frequency

5. The total number of Zeeman components observed in an electronic transition ${}^2D_{5/2} \rightarrow {}^2P_{3/2}$ of an atom in a weak field is

(A) 4 (B) 6 (C) 12 (D) 3

6. In a thermodynamic process, the gas has the equations of state given by $V = V_o[1+\alpha(T-T_o)]$, $\left(\frac{\partial V}{\partial p}\right)_T = 0$ and C_p is a constant. The change in entropy of the gas in the system is

(A) $\Delta S_o = C_p \log T - \alpha V_o p$ (B) Zero

(C) $\Delta S = C_p - \alpha V_o p$ (D) $\Delta S = C_p - \alpha V_o p \log T$.

7. What is the change in rotational constant B when hydrogen is replaced by deuterium in the hydrogen molecule?

(A) 2B (B) 6B (C) B/4 (D) B/2

8. Raman shift is

(A) Independent of frequency of incident radiation, but depends on Scatterer

(B) Independent of Scatterer, but depends on the frequency of incident radiation

(C) Independent of both the frequency of incident radiation and scatterer

(D) Dependent on both the frequency and intensity of incident radiation.

9. What is the nuclear g_N factor for ^{19}F nucleus which has a magnetic moment of 2.6273 μ_N and whose nuclear spin quantum number, I = 1/2?

(A) 5.2546 (B) 1.3136 (C) 10.5092 (D) 0.6568

10. Which one of the following molecules will show a microwave rotational spectrum?

(A) CO_2 (B) SF_6 (C) OCS (D) CH_4

11. If the Lagrangian for a particle in gravitational field is expressed in cylindrical coordinates (r, ϕ, z) as $L = \frac{1}{2}m\dot{z}^2 + \frac{1}{2}mr^2\dot{\phi}^2 - mgz$,, where r is constrained to have a constant value, then

(A) ϕ is a cyclic coordinate (B) ϕ and z are cyclic coordinates

(C) z is a cyclic coordinate (D) ϕ and z are not cyclic coordinates

12. The motion of a free particle is described by the Lagrangian equation $\ddot{x} = 0$. Under the change in the Lagrangian $L \to L + \frac{dF}{dt}$, where $F = x^2 t$, the equation of motion becomes

(A) $\ddot{x} = 2xt$ (B) $\ddot{x} = 2\dot{x}t$ (C) $\ddot{x} = 0$ (D) $\ddot{x} = t^2$

13. Treating the moment of inertia tensor I as a 3 x 3 matrix if the value of its trace is 13 and the values of two of the principal moments of inertia are 4 and 5, then the value of the third principal moment of inertia is

(A) 1 (B) 2 (C) 3 (D) 4

14. In a three dimensional Euclidean space, the number of Eulerian angles required to describe the rotational motion is

(A) 4 (B) 3 (C) 9 (D) 6

15. If the Poisson bracket $\{x, p\} = 1$, the value of $\{x^2, p\}$ is

(A) $2x$ (B) $-2x$ (C) 2 (D) -2

16. In the following transformations from one set of canonical quantities (q, p) to another set of canonical quantities (Q, P), which one is a canonical transformation?

(A) $q = P, p = Q$ (B) $q = -P$; $p = Q$ (C) $q = -P$; $p = -Q$ (D) $q = -Q$; $p = P$

17. For a conservative system, the equilibrium configuration corresponds to the

(A) Lagrangian being zero

(B) Potential energy being zero

(C) Total energy being zero

(D) Gradient of the potential energy being zero

18. The relativistic mass of a particle of rest mass 8 g that is moving with a velocity 1.8×10^8 m/s is equal to

(A) 8 g (B) 10 g (C) 2 g (D) 6.4 g

19. The kinetic energy of a particle of rest mass m_0 moving with a relativistic velocity v is

(A) $\frac{1}{2}m_0v^2$ (B) mc^2 (C) $mc^2 - m_0c^2$ (D) m_0c^2

20. In the Hamilton-Jacobi theory, the new set of canonical quantities Q and P are such that

(A) Only Q is constant (B) Both Q and P are constants

(C) Only P is constant (D) Q and P are not constants

21. Let $\psi(x,t)$ is the position-space wave function corresponding to the state $|\psi(t)\rangle$ of a quantum mechanical particle. If $\langle\psi(t)|\psi(t)\rangle$ is found to be independent of t, this implies that $\left|\psi(x, t)^2\right|$ is

(A) Independent of x (B) Independent of t

(C) A constant (D) Zero

22. A quantum mechanical particle moves freely inside a spherical cavity of radius, R. The wall of the cavity is a perfectly rigid reflector. In this case, the energy levels of the particle are

(A) Degenerate (B) Non-degenerate

(C) Continuous (D) Continuous and non-degenerate

23. If the probability that x lies between x and $x + dx$ is $p(x)\, dx = ae^{-ax}dx$, where $0 < x < \infty$, $a > 0$, then the probability that x lies between x_1 and x_2 $(x_2 > x_1)$ is

(A) $e^{-ax_1} - e^{-ax_2}$ (B) $a(e^{-ax_1} - e^{-ax_2})$ (C) $e^{-a(x_1-x_2)}$ (D) $e^{-a(x_2-x_1)}$

24. The degeneracy of the $n = 2$ level for a three dimensional isotropic harmonic oscillator is

(A) 4 (B) 6 (C) 8 (D) 10

25. Which one of the following relations is true for the Pauli matrices σ_x, σ_y and σ_z?

(A) $\sigma_x\sigma_y = \sigma_y\sigma_x$ (B) $\sigma_x\sigma_y = \sigma_z$ (C) $\sigma_x\sigma_y = -\sigma_z$ (D) $\sigma_x\sigma_y = -\sigma_y\sigma_x$

26. The orbital angular momentum l takes

(A) Only half integer values (B) Both integer and half integer values

(C) Only integer values (D) Continuous values

27. In the Heisenberg picture of quantum mechanics, which is time dependent?

(A) State of the system (B) Operator

(C) Both state and operator (D) Density matrix

28. In the normal Zeeman effect in the hydrogen atom problem, the application of a uniform magnetic field causes the Zeeman splitting of energy levels because

(A) The spherical symmetry is intact

(B) It increases the energy of the system

(C) The spherical symmetry is broken

(D) It decreases the energy of the system

29. For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}mw^2x^2$, the first excited energy eigen state is $\psi(x) = xe^{-ax^2}$. The value of a is

(A) $\frac{1}{4}mw\hbar$ (B) $\frac{1}{2}mw\hbar$ (C) $\frac{1}{2m}\hbar w$ (D) $\frac{1}{2\hbar}mw$

30. If $\hat{p}$ is the density matrix of a pure state and if the state is not normalized to one, then it implies

(A) $\hat{p}^2 = \hat{p}$ (B) $\hat{p}^2 < \hat{p}$ or $\hat{p}^2 > \hat{p}$

(C) $Tr(\hat{p}) = 1$ (D) $Tr(\hat{p}) = 0$

31. The cross-section for emission of bremsstrahlung depends on the target atomic number Z as

(A) Z (B) $Z-1$ (C) $Z-2$ (D) Z^2

32. The collective model predicts about the rotational levels of

(A) Even – Even nuclei (B) Odd – Odd nuclei

(C) Even – Odd nuclei (D) Odd – Even-Odd nuclei

33. The spin-parity assignment of Ca^{41} ($Z = 20$) according to shell model is

(A) $7^+/2$ (B) $7^-/2$ (C) $5^+/2$ (D) $5^-/2$

34. The total scattering cross section of S-wave neutrons when the phase shift is δ_o is

(A) Directly proportional to $\sin\delta_o$ (B) Directly proportional to $\sin^2\delta_o$

(C) Inversely proportional to $\sin\delta_o$ (D) Inversely proportional to $\sin^2\delta_o$

35. The exchange of spin coordinates gives rise to the following potential

(A) Majorana (B) Meson (C) Bartlett (D) Heisenberg

36. The gamma decay $2^+ \to 0^+$ is a

(A) Pure E_2 transition (B) Pure M_2 transition

(C) Mixture of E_2 and M_2 transitions (D) Mixture of E_1 and M_3 transitions

37. The Compton cross section for absorption of gamma rays by matter depends on the atomic number Z as

(A) Z^5 (B) Z^3 (C) $1/Z$ (D) Z

38. Zone plate is an physical object used to illustrate

(A) Fraunhofer diffraction (B) Fresnel diffraction

(C) Interference (D) Polarization

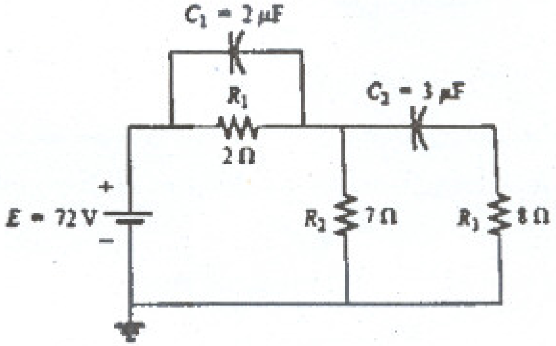
39. The maximum energy of deuteron coming out of a cyclotron accelerator is 20 MeV. The maximum energy of proton that can be obtained from this accelerator is

(A) 40 MeV (B) 30 MeV (C) 20 MeV (D) 10 MeV

40. A particle of mass m and charge q enters a homogeneous and stationary electric field $\vec{E}$ with a velocity v_o perpendicular to the direction of the field. The particles path will be a

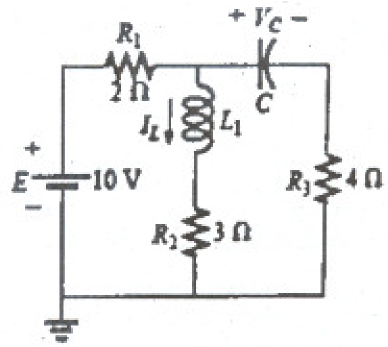
(A) Circle (B) Ellipse (C) Parabola (D) Straight line

41. The energy stored by each capacitor of Figure are



(A) 256μJ, 4704μJ
(B) 256μJ, 470 μJ
(C) 256μJ, 704 μJ
(D) 250μJ, 470 μJ

42. The current I_L and the voltage V_C of Figure are



(A) 6A, 2V (B) 0.5A, 6V (C) 2A, 6V (D) 0.5A, 3V

43. Given the current $i = 6\times10^{-3}\sin(1000t)$, then $t = 2$ ms, the current is

(A) 0.20939 mA
(B) 5.46 mA
(C) 2.052 mA in the negative direction
(D) 5.58 mA

44. If the source current leads the applied voltage, the network is predominantly ________.

(A) Resistive
(B) Inductive
(C) Capacitive
(D) None of the above

45. As the applied frequency increases, the reactance of a inductor ———— and that of a capacitor ————.

(A) Increases linearly, decreases linearly

(B) Increases exponentially, decreases exponentially

(C) Constant, decreases exponentially

(D) Increases linearly, decreases nonlinearly

46. According to Maxwell, the radiation pressure is

(A) Equal to the energy density of the electromagnetic Wave

(B) Equal to the energy density divided by velocity of light

(C) Zero

(D) Proportional to area of illuminating surface

47. In atomic clock, the atom interferometer or in the focusing of atomic beams, which of the following is the basis for its operation?

(A) High purity quartz crystal

(B) Optical laser cooling

(C) Monochromatic laser light

(D) Magnetic lens

48. The absolute refractive index of a medium is proportional to square root of its dielectric constant. This statement is known as

(A) Snell's equation

(B) Fresnel's relation

(C) Dispersion relation

(D) Maxwell's relation

49. Microwaves are electromagnetic waves having

(A) Wavelength ranging in a few μm

(B) Energy in the range of few μW

(C) Frequency ranging from 10^9 Hz to 10^{11} Hz

(D) Frequency ranging from 10^6 Hz to 10^{13} Hz

50. Let $F = (A + BC)(B + CA')$. According to Boolean algebra F is equal to

(A) A

(B) B

(C) C

(D) A'

51. Orthorhombic crystal structure is defined by

(A) $a = b = c, \alpha = \beta = \gamma = 90^\circ$

(B) $a \neq b = c, \alpha = \beta = \gamma = 90^\circ$

(C) $a \neq b \neq c, \alpha = \beta = \gamma = 90^\circ$

(D) $a \neq b \neq c, \alpha = \beta = \gamma \neq 90^\circ$

52. According to Fermat's principle, light, while traveling from one point A to another point B across several medium with different refractive indices, traverses the route having the

(A) The smallest optical path length from A to B

(B) Curved but still minimum distance between A and B

(C) Lowest refractive index of the medium

(D) Straight line path from A and B

53. The Miller Indices of the planes parallel to a-c plane is

(A) 100 (B) 010 (C) 001 (D) 111

54. The number of atoms per unit cell of the reciprocal of bcc structure is

(A) 1 (B) 2 (C) 3 (D) 4

55. Hydrogen bond is found in

(A) H_2 molecule (B) HCl molecule

(C) H_2O molecule (D) HF molecule

56. Which degeneracy is not permitted for the rotational symmetry in a crystal

(A) 2 fold (B) 3 fold (C) 4 fold (D) 5 fold

57. The potential energy between two atoms in a molecule is given by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^{6}}$ where a and b are positive constants and x is the distance between two atoms in a molecule. The molecule will be in stable equilibrium if

(A) $x=0$ (B) $x=\left(\frac{a}{2b}\right)^{1/6}$ (C) $x=\left(\frac{11a}{5b}\right)^{1/12}$ (D) $x=\left(\frac{2a}{b}\right)^{1/6}$

58. The lattice constant of Fe with bcc structure, density 7860 kg/m^3 and atomic weight 55.85 is

(A) 0.143 nm (B) 0.286 nm (C) 1.43 nm (D) 2.86 nm

59. For the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\exp(A)$ is

(A) $\begin{bmatrix} e & e \\ e & e \end{bmatrix}$ (B) $\begin{bmatrix} e & 0 \\ 0 & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & e \\ e^2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix}$

60. The process of contraction of a tensor reduces its rank by

(A) 0 (B) 1 (C) 2 (D) 3

61. The value of the integral $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$ using residue theorem is

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{2}$ (C) 2π (D) Zero

62. Let $\mathbf{F}$ denote the Fourier transform and let $\mathbf{F}[f(t) = g(w)]$. Then, $\mathbf{F}[f(t)\cos(wt)]$ is equal to

(A) $\frac{1}{2}g(w-a) - \frac{1}{2}(w+a)$ (B) $\frac{1}{2}g(w-a) + \frac{1}{2}g(w+a)$

(C) $\frac{1}{2}g(w-a)$ (D) $g(w+a)$

63. Let J_o denote the Bessel function. Then, the Laplace transform of J_o is

(A) $\frac{1}{\sqrt{s^2+1}}$ (B) $\sqrt{s^2+1}$ (C) $\sqrt{s^2-1}$ (D) $\frac{1}{\sqrt{s^2-1}}$

64. Dirac delta function is used when a function exists with

(A) Zero values in a very short interval

(B) Non-zero values in a very short interval

(C) Non-zero values in a long interval

(D) Infinite values in a long interval

65. The value of the integral $\oint (z^2 - 2z - 3)\,dz$, where the contour is the circle with $|z| = 2$ is

(A) $2\pi i$ (B) $\frac{1}{2\pi i}$ (C) 2 (D) Zero

66. The components of an ordinary vector in an N-dimensional space are the components of

(A) Vector (B) Mixed tensor

(C) Covariant tensor of rank I (D) Contravariant tensor of rank 1

67. The eigenvalues of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ are

(A) $e^{-i\theta}, e^{-i\theta}$ (B) $e^{i\theta}, e^{i\theta}$ (C) $e^{+i\theta}, e^{-i\theta}$ (D) 0, 1

68. Which of these equation is not a Maxwell's equation for a static electromagnetic field in a linear homogeneous media?

(A) $\vec{\nabla}\cdot\vec{B}=0$ (B) $\vec{\nabla}\cdot\vec{D}=0$ (C) $\oint\vec{B}\cdot\vec{dl}=\mu_0 I$ (D) $\nabla^2\vec{A}=\mu_0\vec{J}$

69. For a linear medium Poisson equation becomes Laplace's equation when

(A) Charge density is constant

(B) Charge density is zero

(C) Charge density is uniform

(D) Charge density is spherically symmetric

70. Two identical coils carry the same current I but in the opposite direction, the magnitude of the magnetic field B at point on the axis midway between the coils is

(A) Zero

(B) Twice that is produced by one coil

(C) Same as that produced by one coil

(D) Half as that produced by one coil

71. The SI unit of magnetic charge is

(A) $A.m^2$ (B) Coulomb (C) A.m (D) Ampere

72. Let a fluid has compressibility k and a density ρ and Hook's law holds good for the fluid. Suppose a one-dimensional wave travels in the fluid, then the wave will be

(A) A non-dispersive wave
(B) A dispersive wave
(C) A nonlinear wave
(D) A solitary wave

73. Which of the following appliances do not operate in microwave region?

(A) Microwave oven
(B) Satellite Television
(C) Police radar
(D) AM Radio

74. A light pulse propagates through a fiber cable with attenuation of 0.25 dB/Km, the power of pulse is reduced by 40% after a distance of

(A) 8.874 km
(B) 88.74 km
(C) 80000 km
(D) 0.25 km

75. The skin depth in copper for EM wave of frequency 10Hz and 10^8Hz is respectively

(A) Same for both frequencies
(B) Skin depth at 10 Hz is higher
(C) Skin depth at 10^8Hz is higher
(D) Zero for copper

76. Which of the following is not a source of static magnetic field?

(A) A dc current in wire
(B) A permanent magnet
(C) A charged disc rotating with constant speed
(D) An accelerating charge

77. In cylindrical coordinates the equation $\frac{\partial^2 \psi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \psi}{\partial z^2} + 10 = 0$ is called

(A) Helmholtz Equation
(B) Laplace's equation
(C) Poisson's Equation
(D) Lorentz Equation

78. A particle of mass m moving in one dimension is assumed to have any velocity in the range $[-v_0, +v_0]$ with equal probability. So we have the probability density $P(v) = \frac{1}{2v_0}$. The probability density for kinetic energy E goes as

(A) E

(B) $\sqrt{E}$

(C) Independent of E

(D) $\frac{1}{\sqrt{E}}$

79. Consider a system of f degrees of freedom of which there are $f/3$ cyclic coordinates. Each non cyclic coordinate appears quadratic in Hamiltonian. The internal energy of the system at temperature T is

(A) $\frac{f}{2}kT$ (B) fkT (C) $\frac{f}{3}kT$ (D) $\frac{2f}{3}kT$

80. Let C_p and C_v be specific heat capacities at constant pressure and volume respectively. Then, for ideal gas consisting of relativistic particles, $\frac{C_p}{C_v}$ is

(A) 5/3 (B) 3/5 (C) 3/4 (D) 4/3

81. Pressure exerted by photon gas is related to energy density u as

(A) $u/3$ (B) $2u/3$ (C) $3u/5$ (D) $5u/3$

82. Mean square fluctuations in energy of a canonical ensemble is related to

(A) Entropy

(B) Gibb's free energy

(C) Specific heat

(D) Chemical potential

83. Density of states at E for free electrons in metals is proportional to E^{α}. The values of α in 1, 2 and 3 dimensions respectively are

(A) $-1/2, 0, 1/2$ (B) $1/2, 0, 1/2$ (C) $-1/2, 1/2, 0$ (D) $1/2, 1/2, 0$

84. In the reaction called nitrogen fixation given by $N_2 + 3H_2 \leftrightarrow 2NH_3$ if the total pressure is increased, what happens to NH_3?

(A) Production of NH_3 decreases

(B) Production of NH_3 increases

(C) Production of NH_3 remains unchanged

(D) NH_3 decomposes

85. Two fermions occupy three states of energies 0, ε and 2ε. The ground and first excited state energies respectively are

(A) 0 and ε (B) ε and 2ε (C) ε ad 3ε (D) 0 and 2ε

86. Consider a system at temperature T and number density n. Quantum statistics is necessary if $T < n^{\alpha}$, where α is

(A) 1/3 (B) 3/2 (C) 1/2 (D) 2/3

87. Specific heat of two dimensional crystalline solids at low temperature is proportional to

(A) T^3 (B) T^1 (C) T^2 (D) T^0

88. Consider the ordinary differential equation (ODE) $y'' + x^2 y = 0$ with initial conditions $y(0) = A$ and $y'(0) = B$. If we apply Laplace transform to this differential equation, we get

(A) A second order ODE with constant coefficients

(B) A second order ODE with variable coefficients

(C) A first order ODE with constant coefficients

(D) A linear or non-linear algebraic equations

89. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then the value of α is

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 5

90. If $f(x)$ is differentiable and is a strictly increasing function, then $\lim_{x \to 0} \frac{f(x)^2 - f(x)}{f(x) - f(0)}$ is equal to

(A) 1 (B) 0 (C) 2 (D) -1

91. Let L^{-1} denote the inverse Laplace transform. Then, $L^{-1}\left[\frac{2as}{(s^2 + a^2)^2}\right]$ is

(A) $t\sin(at)$ (B) $at\sin(at)$ (C) $t - \sin(at)$ (D) $-t\sin(at)$

92. For the differential equation $2x^2y' + x(2x+1)y' - y = 0$, the point $x = 0$ is

(A) A regular singular point

(B) Not a singular point

(C) A singular point but not regular

(D) An isolated singular point

93. Suppose that one solution y_1 of the differential equation $y'' + p(x)y' + q(x)y = 0$ is known to us. Then, the second linearly independent solution can be found by using

(A) $y_2 = \int \frac{1}{y_1^2} \exp\left(-\int p(x)dx\right)dx$

(B) $y_2 = \int \frac{1}{y_1^2} \exp\left(\int p(x)dx\right)dx$

(C) $y_2 = \int \frac{1}{y_1} \exp\left(-\int p(x)dx\right)dx$

(D) $y_2 = \exp\left(-\int p(x)dx\right)$

94. The integrating factor for the differential equation $(3y^2 + 2xy)dx - (2xy + x^2)dy = 0$ is

(A) $\exp\left(\frac{-1}{x+y}\right)$

(B) Integrating factor is not obtainable

(C) $\frac{1}{xy(x+y)}$

(D) e^{-xy}

95. Let $y = cx + \frac{1}{c}$, $c \neq 0$. If we obtain a suitable ordinary differential equation (ODE) by eliminating the constant c, then we get,

(A) First order second degree nonlinear ODE

(B) First order second degree linear ODE

(C) First order first degree linear ODE

(D) Second order first degree linear ODE

96. We are interested to change the variable from x to t using $x = e^t$. If we apply this transformation to the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$, we get

(A) $\frac{d^2y}{dt^2} - 4y = 0$

(B) $\frac{d^2y}{dt^2} + e^{-t}\frac{dy}{dt} - 4y = 0$

(C) $\frac{d^2y}{dt^2} + e^{-2t}\frac{dy}{dt} + 4y = 0$

(D) $\frac{d^2y}{dt^2} + e^{t}\frac{dy}{dt} + 4y = 0$

97. The Bessel function of order υ and first kind is $J_{\pm\upsilon}(x)$ and that of second kind is $Y_{\upsilon}(x)$. We know that Bessel functions are solutions of the Bessel differential equation $x^2y'' + xy' + (x^2 - \upsilon^2)y = 0$. Then, the general solution of the equation $x^2y'' + xy'\left(x^2 - \frac{1}{16}\right)y = 0$ is given by

(A) $y(x) = AJ_{1/4}(x) + BY_{-1/4}(x)$

(B) $y(x) = AJ_{1/4}(x) + BJ_{-1/4}(x) + CY_{1/4}(x)$

(C) $y(x) = AJ_{1/4}(x) + BY_{1/4}(x)$

(D) $y(x) = AJ_{1/4}(x) + BJ_{-1/4}(x)$

98. Let **F** denote Fourier transform and let $\mathbf{F}[f(t)] = \mathbf{F}(w)$. Select the <u>wrong</u> statement from the options.

(A) $\mathbf{F}[\mathbf{F}[f(t)]] = 2\pi f(w)$ is the symmetry property of Fourier transform

(B) $\mathbf{F}[\exp(iw_0t)f(t)] = \mathbf{F}(w + w_0)$ is the w-shifting property

(C) $\mathbf{F}\left[\int_{-\infty}^{t} f(\tau)dT\right] = \frac{1}{iw}\mathbf{F}(w)$ is the integration property of Fourier transform

(D) $\mathbf{F}[f^{(n)}(t)] = (iw)^n \mathbf{F}(w)$ is the derivative property of Fourier transform

99. If we apply Fourier transform with respect to x to the partial differential equation $\frac{\partial u(x,t)}{dt} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$, we get,

(A) A second order linear ODE in w and t

(B) A first order linear ODE in w and t

(C) A first order nonlinear ODE in w and t

(D) An algebraic equation in w and t

100. How many boundary conditions are required to solve the <u>second order</u> partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$?

(A) 2 only

(B) Minimum 2 and maximum 4

(C) Minimum 4 and maximum any

(D) 4 only